STATICS--AN INVESTIGATION OF FORCES

Two areas of study to investigate forces

A. Statics—where the forces acting on a material are balanced so that the material is either stationary or in uniform motion. For fluid materials, the study of static forces is called hydrostatics.

Examples:

1) A book lying on a table (statics)
2) Water being held behind a dam (hydrostatics)

B. Dynamics—where the forces acting on a material are not balanced and the material accelerates. For fluid materials, the study of dynamic forces is called hydrodynamics or fluid dynamics.

Examples:

1) A roller coaster executing a loop (dynamics)
2) The flow of water coming from a garden hose (hydrodynamics)

Note: Statics is by far the easier topic

Physical processes and variables possess one of three different attributes

A. Scalars—variables whose values are expressible purely as a magnitude or quantity. Example: weight, pressure, speed.

Representation: P, r, θ.

B. Vectors—variables whose values require specifications of both magnitude and direction. Example: position, velocity, force. A vector requires three scalar values, one corresponding to each of the three spatial directions. Subscripts are often used to denote direction.

Representation: v_i , $\underline{a}$, $\boldsymbol{F}$.

C. Tensors—variables whose values are collections of vectors. Example: the stress tensor of a solid subjected to load, the deformation tensor of a block of ice under pressure. Multiple subscripts are often used to denote combination directions.

Representation: C_{ijkl}, S_{ij}, $\underline{\underline{\tau}}$.

A tensor with two subscripts is called a "second order" tensor and requires an array of nine scalar values to represent it.

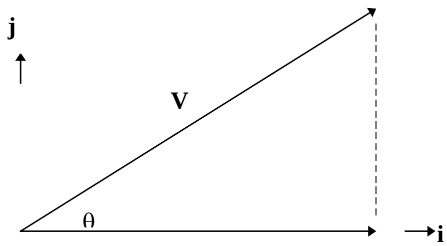
Note: Scalars and vectors can be considered as zeroth order and first order tensors.

Vectors

A. Properties
   1. Magnitude
   2. Direction
   3. Act along the line of their direction
   4. No fixed origin

B. Visualization
   1. An arrow having a direction and a length
      a) length implies the total magnitude of the vector
      b) arrow orientation implies the direction of the vector
   2. The negative of a vector is the same-oriented, same-magnitude arrow pointing in the opposite direction.

C. Projections, components
   1. The magnitude of a vector in any given direction is the geometric projection of that vector in that direction.



projection of **V** in **i**-direction

In cartesian coordinates:

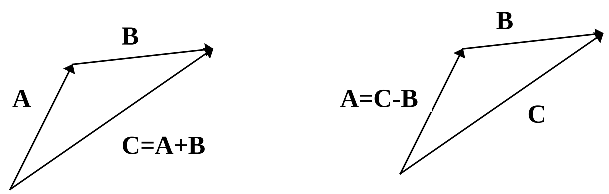
$V_x = |\mathbf{V}| \cos\theta$

$V_y = |\mathbf{V}| \sin\theta$

   2. A vector **V** can be completely represented as (or decomposed into) a sum of vectors in any three independent directions. For the conventional x,y,z cartesian system, one representation is $a\,\mathbf{i} + b\,\mathbf{j} + c\,\mathbf{k}$. *a,b,c* are the magnitudes of the unit vectors **i**, **j**, **k** in the x, y, z directions. The magnitude of the vector is $|\mathbf{V}| = \sqrt{a^2 + b^2 + c^2}$.

3. Vectors add and subtract.

a) Graphically



b) Algebraically

$\mathbf{A} = a_1\,\mathbf{i} + a_2\,\mathbf{j} + a_3\,\mathbf{k}$ $\mathbf{B} = b_1\,\mathbf{i} + b_2\,\mathbf{j} + b_3\,\mathbf{k}$

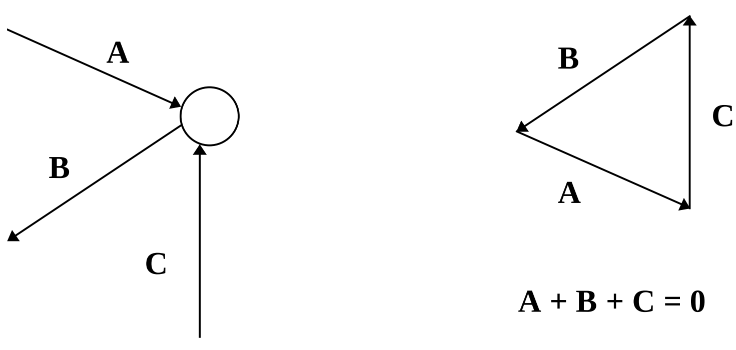
$\mathbf{C} = \mathbf{A} + \mathbf{B} = (a_1 + b_1)\,\mathbf{i} + (a_2 + b_2)\,\mathbf{j} + (a_3 + b_3)\,\mathbf{k}$

STATICS

A. Two conditions for static equilibrium:

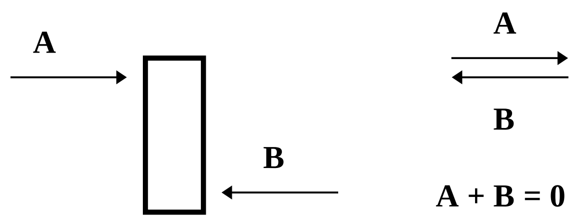
1. Note: Visualize all the forces on a body by creating a **free-body diagram**—a picture which shows all the external and reactive forces on the body. The following sketches are free-body diagrams.

1. On an object, $\sum F = 0$. And, since force is a vector, this condition implies that $\sum F_x = 0$, $\sum F_y = 0$, $\sum F_z = 0$ individually.



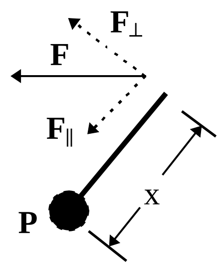
A + B + C = 0

But, sum of forces equal zero is not sufficient:



2. About any point on an object, the sum of the moments is zero.

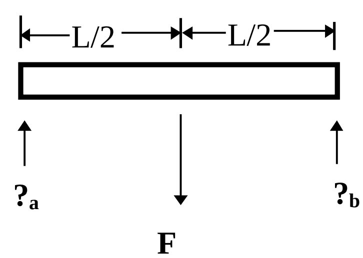
Moment M (or torque τ): a scalar quantity which describes how much "twist" a force exerts at a point P. It's calculated as the magnitude of the force times the "moment arm" or perpendicular distance that the forces acts from the point. (Clockwise moments are positive; counterclockwise are negative.)



$M_P = |\mathbf{F}_\perp| * x$

B. Examples:

1. If a beam supported at its endpoints is given a load **F** at its midpoint, what are the supporting forces at the endpoints?



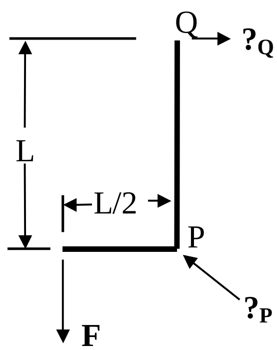
a. Sum of the moments about **a** :

$|\mathbf{F}|\ L/2 - |\mathbf{?_b}|\ L = 0 \Rightarrow |\mathbf{?_b}| = |\mathbf{F}| / 2$

$\therefore \mathbf{?_b} = -\ \mathbf{F}/2$

b. Sum of forces: $\mathbf{F} + \mathbf{?_a} + \mathbf{?_b} = 0 \quad \Rightarrow \quad \mathbf{?_a} = -\ \mathbf{F}/2$

2. An "L" lever is pinned at the center P and holds load **F** at the end of its shorter leg. What force is required at Q to hold the load? What is the force on the pin at P holding the lever?



a) Moments: $|?_Q|\ L - |\mathbf{F}|\ L/2 = 0$

$\therefore |?_Q| = 1/2\ |\mathbf{F}|$

b) forces in **x**-direction: $?_{px} = -\ ?_Q$
forces in y-direction: $?_{py} = -\ \mathbf{F}$

$\therefore ?_P = -\ 1/2\ |F|\ \mathbf{i} + |F|\ \mathbf{j}$

Trusses:

A. A truss consisting of tension and compression members held together by hinge or pin joints is called a simple truss.

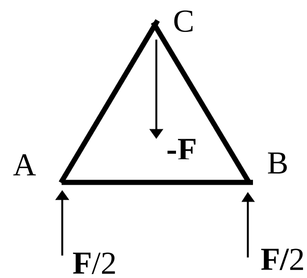
B. Assumptions for analyzing forces of simple trusses:

1. Joints are assumed to be frictionless, so forces can only be transmitted in the direction of the members.
2. Members are assumed to be massless.
3. Loads can be applied only at joints (or nodes).
4. Members are assumed to be perfectly rigid.

C. For static equilibrium, a truss must meet the following two conditions:

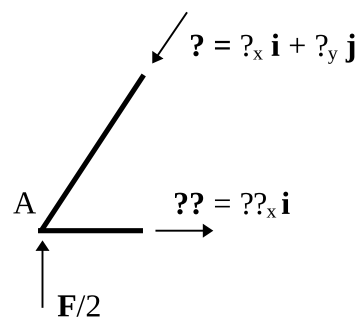
1. The forces at each joint (or nodal point) must sum to zero.
2. The moments about any joint must sum to zero.

D. Analyzing forces within a simple truss ABC with equal-length sides, load **F** at node C, and support at nodes A and B:



Sum of the moments about A yield the support forces equal to **F**/2

1. Analysis of forces at node A:



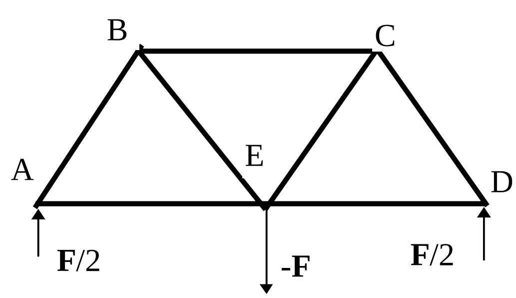
a) $?_y = -|\mathbf{F}|/2$ because A is in equilibrium in the **y**-direction.

b) $?_y = |\mathbf{?}|\sin 60^o \Rightarrow |\mathbf{?}| = |\mathbf{F}|/\sqrt{3}$ because **?** lies along AB.

c) $??_x = |\mathbf{?}| \cos 60^o = |\mathbf{F}|/(2\sqrt{3})$ because A is in equilibrium in the **x**-direction.

As a result, member AC is in compression; member AB is in tension.

E. Analyzing a more complicated truss, with a load at E and supports at A and D:



a) Analyze forces at A. This yields forces along AB and AE.

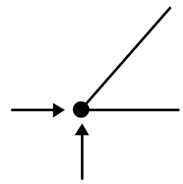
b) Analyze forces at B. This yields forces along BC and BE.

c) Analyze forces at C. Analyze forces at E.

d) (If bridge is symmetric, forces along CD, DE, CE are the same as those along AB, AE, BE, respectively.

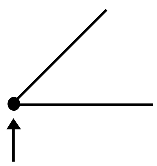
F. Supports at nodal points

1. Fixed support—can react to load in both x and y directions; treated as two supports



support in two directions:
"fixed" node

2. Rolling support—can react to load in only one direction, e.g., y; can support no load in x direction.

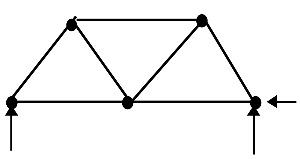


support in one direction:
"rolling" node

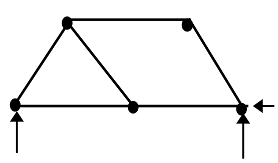
G. Conditions of static determinacy,. stability of trusses

1. Let: m = number of members in the truss
   r = number of external supports for the truss
   j = number of nodes in the truss

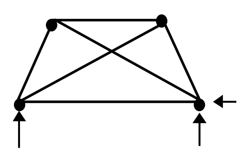
   a) m + r = 2j is statically determinate
   b) m + r < 2j is statically unstable
   c) m + r > 2j is statically indeterminate



m = 7, r = 3, j = 5
statically determinate



m = 6, r = 3, j = 5
statically unstable

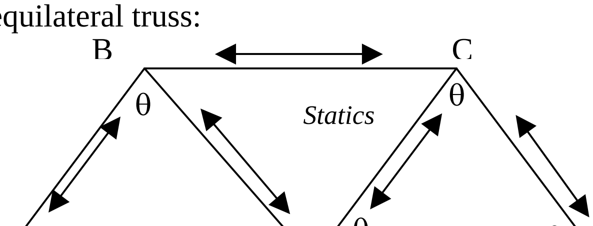


m = 6, r = 3, j = 4
statically indeterminate

2. m + r = 2j is a necessary, but not sufficient condition for static determinacy

3. Statically indeterminate structures can be analyzed using more advanced methods. These methods require knowledge of E and I for each member.

Analysis for an equilateral truss:



Assume forces in truss are as indicated. Then the forces at each node are as follows:

At A: $S1x - F_{AE} - F_{AB}\cos\theta = 0$
$S1y - F_{AB}\sin\theta = 0$

At B: $F_{AB}\cos\theta - F_{BE}\cos\theta - F_{BC} = 0$
$F_{AB}\sin\theta + F_{BE}\sin\theta = 0$

At C: $F_{BC} + F_{CE}\cos\theta - F_{CD}\cos\theta = 0$
$F_{CE}\sin\theta + F_{CD}\sin\theta = 0$

At D: $F_{DE} + F_{CD}\cos\theta = 0$
$S2 - F_{CD}\sin\theta = 0$

At E: $F_{AE} - F_{DE} + F_{BE}\cos\theta - F_{CE}\cos\theta = 0$
$-F_{BE}\sin\theta - F_{CE}\sin\theta - L = 0$

Then these equations can be put into matrix form as:

$$\begin{pmatrix} 1 & 0 & -\cos & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -\sin & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sin & 0 & \sin & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \cos & 0 & -\cos & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & \cos & -\cos & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sin & \sin & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\sin & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \cos & 1 & 0 \\ 0 & 0 & 0 & 1 & \cos & 0 & \cos & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -\sin & 0 & -\sin & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} S1x \\ S1y \\ F_{AB} \\ F_{AE} \\ F_{BE} \\ F_{BC} \\ F_{CE} \\ F_{CD} \\ F_{DE} \\ S2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ L \end{pmatrix}$$