

MATERIALS

Why do things break?
 Why are some materials stronger than others?
 Why is steel tough?
 Why is glass brittle?

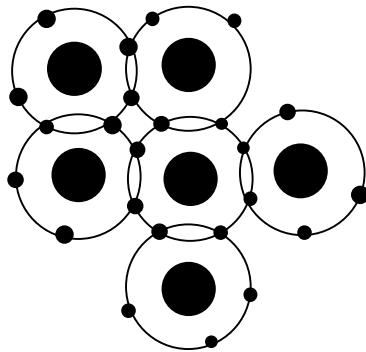
What is toughness? strength? brittleness?

Elemental material—atoms:

- A. Composition
 - a) Nucleus: protons (+), neutrons (0)
 - b) Electrons (-)
- B. Neutral charge, i.e., # electrons = # protons
- C. Electrons orbit about nucleus in shells; # of electrons/shell $2N^2$, where N is shell number.
- D. Reactivity with other atoms depends on # of electrons in outermost shell: 8 is least reactive.
- E. Electrons in outermost shell called “valence” electrons
- F. Inert He, Ne, Ar, Kr, Xe, Rn have 8 electrons in shells 1-6, respectively (except for He).

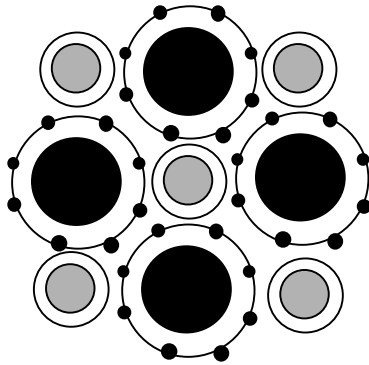
Solids

- A. Form
 1. Crystals--molecules attracted to one another try to cohere in a systematic way, minimizing volume. But perfect "packing" is usually partially interrupted by viscosity.
 2. Glasses and ceramics--materials whose high viscosity at the liquid-solid point prevents crystallization. These materials are usually "amorphous".
 3. Polymers--materials built up of long chains of simple molecular structures. Characteristics of plastics and living things.
 4. Elastomers--long-chain polymers which fold or coil. Natural and artificial rubber. Enormous extensions associated with folding and unfolding of chains.
- B. Held together by chemical, physical bonds
 1. Bonds holding atoms together
 - a) Covalent bonding--two atoms share electrons. Very strong and rigid. Found in organic molecules and sometimes ceramics. Strongly directional.



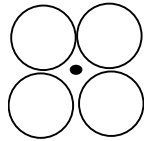
Example: carbon atoms—4 valence electrons

b) Ionic bonding—one atom gives up an electron to become a “cation”; the other gets that electron to become an “ion”. These now-charged atoms are attracted by electrostatic forces. Omnidirectional.

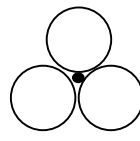


Example: Na (+) (small) and Cl (-)(large)

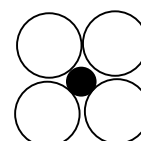
Packing: as close as possible.



NO



YES

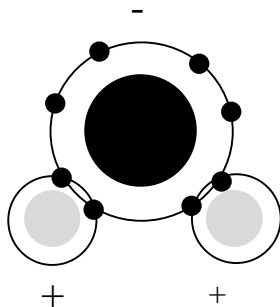


YES

c) Metallic bonds--hold metals and alloys together. Allows for dense packing of atoms, hence metals are heavy. Outer orbit gives up one electron (on average) which is free to roam. Resulting metal ions (+1) are held together by “sea” of electrons. Good electrical conductivity. Omnidirectional.

2. Bonds holding molecules together

a) Hydrogen bonds--organic compounds often held together by charged -OH (hydroxyl) groups. Directional. Due to distribution of charge on molecule. Weak.



Example: H₂O Covalent bonding (angle of 104°) ⇒ “polar molecule”

b) Van der Waal forces--forces arising from surface differences across molecules. Like polar molecules, but not fixed in direction. Very weak.

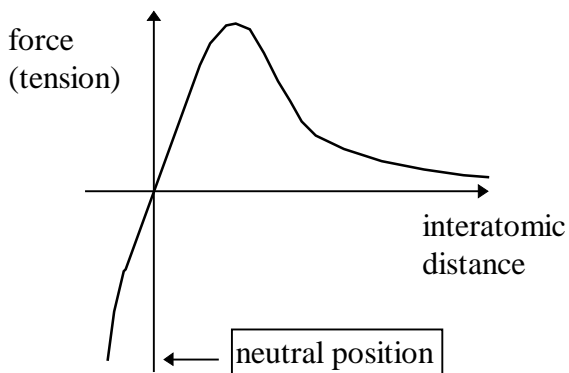
- C. Atoms in equilibrium with interatomic forces at fixed distances from other atoms; closer or farther produces restoring forces; (think of a spring)
- D. Pushing on solid causes deformation (strain) which generates reactive force (stress)

Stress-- σ load per unit area. units: p.s.i. or MegaNewtons/ m^2 .

Strain-- ϵ deformation per unit length units: dimensionless

Hooke's Law

- A. Robert Hooke, 1679 "As the extension, so the force", i.e., stress is proportional to strain
- B. Hooke's law: an approximation of the relationship between the deformation of molecules and interatomic forces.



Solid behavior

- A. Elastic--for most materials and for small deformations, loading and unloading returns material to original length--can be done repeatedly, e.g., a watch spring.
- B. Plastic--larger deformations are not reversible when "elastic limit" is exceeded. Some materials are almost purely plastic, e.g., putty.

Elastic solids

- A. Young's modulus: Thomas Young (1800?) realized that $E = \text{stress/strain} = \sigma/\epsilon = \text{constant}$ described flexibility and was a property of the material. This is also a definition of stiffness.
- B. E has units of stress. Think of E as the stress required to deform a solid by 100%. (Most solids will fail at an extension of about 1%, so this is usually hypothetical).
- C. Range of E in materials is enormous $E(\text{rubber})=0.001 \cdot 10^6$ p.s.i. $E(\text{diamond})=170 \cdot 10^6$ p.s.i. $E(\text{spaghetti}) \approx 0.7 \cdot 10^6$ p.s.i.

Material strength

A. Tensile strength

How hard a pull required to break material bonds?

steel piano wire = 450,000 p.s.i.
 aluminum = 10,000 p.s.i.
 concrete = 600 p.s.i.

B. Compression strength

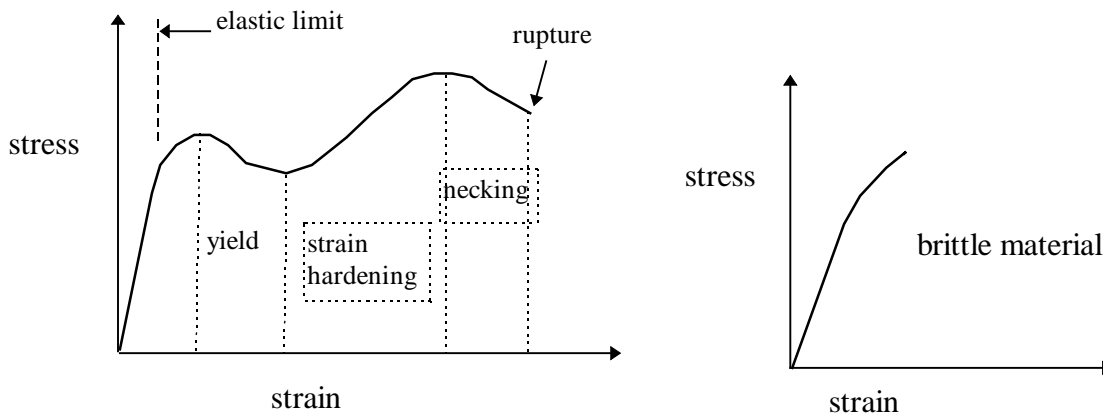
1. Difficult to answer, because materials fail in compression in many ways depending on their geometry and support
 - a) buckling--hollow cylinders, e.g., tin can
 - b) bending--long rod or panel
 - c) shattering--heavily loaded glass

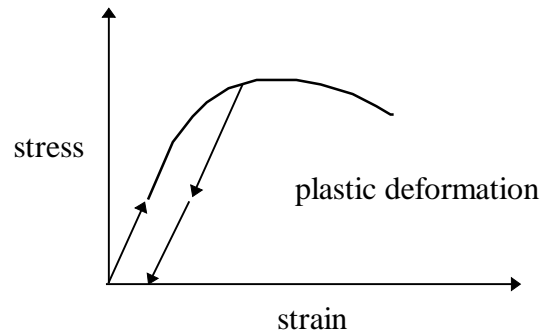
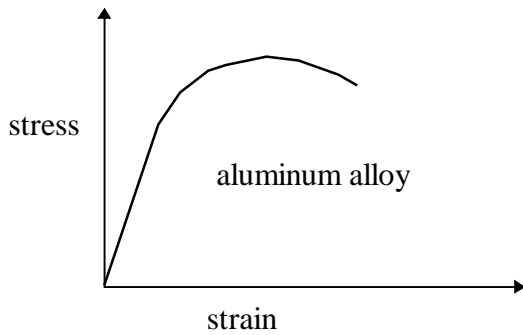
C. No relation between compressive and tensile strength in part because distinction between a material and a structure is often not clear. e.g., what is a brick? or concrete?

D. Other strengths

1. Shear strength--rotating axles fail because their shear strengths were exceeded
2. Ultimate tensile strength--maximum possible load without failure
3. Yield strength--load required to cross line from elastic to plastic deformation

E. Stress-strain diagrams characterizing materials





F. Terms associated with material properties

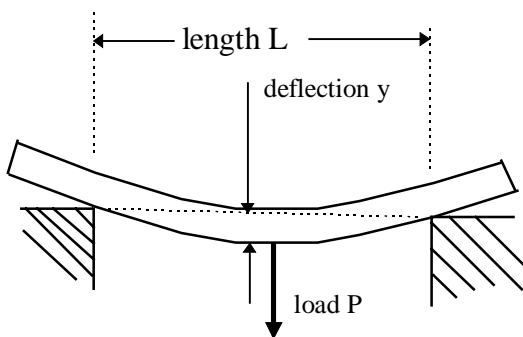
1. Hardness--resistance to scratching and denting.
2. Malleability--ability to deform under rolling or hammering without fracture.
3. Toughness--ability to absorb energy, e.g., a blow from a hammer. Area under stress-strain curve is a measure of toughness
4. Ductility--ability to deform under tensile load without rupture; high percentage elongation and percent reduction of area indicate ductility
5. Brittleness--material failure with little deformation; low percent elongation and percent area reduction.
6. Elasticity--ability to return to original shape and size when unloaded
7. Plasticity--ability to deform non-elastically without rupture
8. Stiffness--ability to resist deformation; proportional to Young's modulus E (psi)
 $E = \text{stress/strain}$ (slope of linear portion of stress/strain curve).

G. Material testing

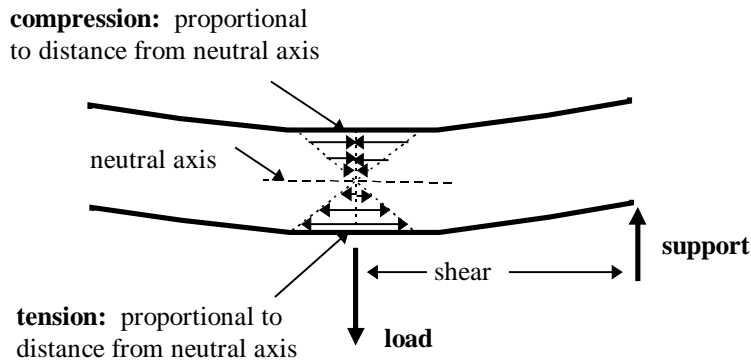
1. Tensile strength

- a) Usually tested by controlling extension (strain) and measuring resulting load (stress*area), i.e., independent variable is strain, dependent variable is stress
- b) Can also be determined by subjecting material to a predetermined load and measuring elongation, i.e., independent variable is stress, dependent variable is strain

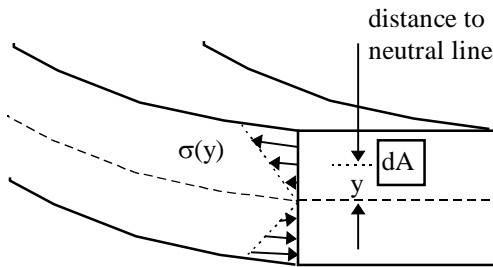
2. Bending



a) Stress/strain in bending



b) Restoring moment due to internal stresses



Restoring moment = (moment arm about neutral line) x (force) = $\sum y\sigma(y) dA$.

But, σ is proportional to strain ϵ , and strain varies linearly with distance to the neutral line. Therefore, $\sigma = y \sigma_{\max}$, where σ_{\max} is the stress at the maximum distance from the neutral line. So,

Restoring moment = $\sigma_{\max} \sum y^2 dA = \sigma_{\max} I$, where I is the moment of inertia of the cross section of the beam about the neutral axis.

Moment of inertia depends on cross-section geometry and has units L^4 .

i) cylindrical rod: $I = \frac{\pi r^4}{4}$

ii) square rod: $I = \frac{s^4}{12}$

iii) moments can be calculated one component at a time, e.g.,

moment of a hollow cylinder: $I = \frac{\pi r_2^4}{4} - \frac{\pi r_1^4}{4}$, where r_2 and r_1 are the outer and inner radii of the cylinder, respectively.

c) Bending deflection from dimensional arguments—assuming the following contributing variables:

y	deflection (L)
P	load (MLT^{-2})
I	moment of inertia (L^4)
E	Young's modulus ($ML^{-1}T^{-2}$)
L	length (L)

Five variables, three primitive dimensions = two dimensionless groups:

$$0 = y^a P^b I^c E^d L^e = L^a (MLT^{-2})^b L^{4c} (ML^{-1}T^{-2})^d L^e$$

$$M: b + d = 0$$

$$L: a + b + 4c - d + e = 0$$

$$T: -2b - 2d = 0$$

Begin with $c = 1, d = 1, a = 1 \Rightarrow b = -1, e = -3$

$$\pi_1 = \frac{EIy}{PL^3} = f(\pi_2)$$

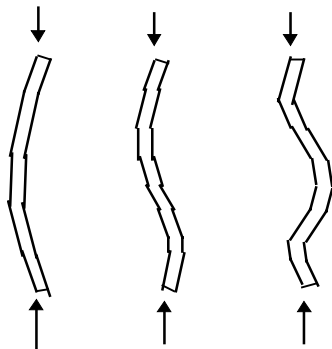
For second group, try $c = 0, d = 0, e = 1 \Rightarrow b = 0, a = -1$

$$\pi_2 = \frac{L}{y} \Rightarrow \frac{EIy}{PL^3} f\left(\frac{L}{y}\right)$$

b) Actual relationship is $E = \frac{PL^3}{48Iy}$. Note: One can use this expression to obtain E from an experiment, or to predict y if E is known for the material.

3. Compressive strength of material

- Under compression a beam will fail either by crushing or buckling, depending on the material and L/d; e.g., wood will crush if L/d < 10 and will buckle if L/d > 10 (approximately).
- Crushing: atomic bonds begin to fail, inducing increased local stresses, which cause more bonds to fail.
- Buckling: complicated, because there are many modes



1st, 2nd, and 3rd order bending modes. Lowest order is most likely to occur

d) Buckling strength calculations—Euler buckling load P_c :

P	load (MLT^{-2})
I	moment of inertia (L^4)
E	Young's modulus ($ML^{-1}T^{-2}$)
L	length (L)

Four variables, three primitive dimensions = one dimensionless group:

$$0 = P^a I^b E^c L^d = (MLT^{-2})^a L^{4b} (ML^{-1}T^{-2})^c L^d$$

$$M: a + c = 0$$

$$L: a + 4b - c + d = 0$$

$$T: -2a - 2c = 0$$

Begin with $a = 1, b = -1 \Rightarrow c = -1, d = 2$; get $\pi_1 = \frac{PL^2}{EI}$, or $P \propto \frac{EI}{L^2}$.

For a cylindrical rod, Euler buckling load $P = P_c \propto \frac{Er^4}{L^2}$.

(This equation can be deduced from the beam deflection equation which determines deflection y

at a position x from one end in terms of load P : $EI \frac{d^2 y}{dx^2} = -P_c y$)