270.307 Combining Measurements with Models
Mid-Term Test, Fall 2004

Name:

Answer FOUR questions and attach this question sheet to the top of your answers. The expected number of marks is shown after each question.

1. (a) What is the χ^2 (chi-squared) probability-distribution function? (8 marks)

   (b) State and explain the central limit theorem discussed in class (a formal, rigorous, definition is not required). (8 marks)

   (c) What can be deduced about the χ^2 probability-distribution function from the central limit theorem? (4 marks)

2. If you were given a list of 100 numbers how would you test to see if they came from an independent, random Gaussian process with zero mean and variance 1? (20 marks)

3. (a) *Retrograde motion of the planets* and *brightness variation of the planets* are key phenomena that any model of the solar system must account for. Explain the two terms in italics. (6 marks)

   (b) How does the Ptolemaic model account for the two terms in italics in (a)? (7 marks)

   (c) How does the Copernican model account for the two terms in italics in (a)? (7 marks)

4. Explain the steps in cubic interpolation. Identify the data, model, and parameters in the problem using an example of your choice. Outline how the problem is solved (but do not give an explicit solution). (20 marks)

5. In solving regression problems we often use the formula: $\tilde{\mathbf{x}} = (\mathbf{E^T E})^{-\mathbf{1}}\mathbf{E^T y}$. Illustrating your answers with a practical example,

   (a) Explain the terms $\tilde{\mathbf{x}}$, $\mathbf{E}$, $\mathbf{y}$, superscript T, and superscript -1. (10 marks)

   (b) State the problem this formula solves. (5 marks)

   (c) Explain the origin of this formula (a rigorous derivation is not required). (5 marks)