270.307 Combining Measurements with Models
Final Test, Fall 2005

Name:

Answer ALL questions and attach this question sheet to the top of your answers. The expected number of marks is shown after each question. You may use your class notes and other sources to answer the test provided you cite all references. Do not discuss the test with anyone. Have a good holiday!

I confirm that I have completed this exam without unauthorized assistance from any person, materials, or device:

Signed: Date:

1. Imagine you are given a sequence of daily-average air temperature measurements at Olin Hall for the period 1900–present.

   (a) Describe how you would calculate the daily-average air temperature anomalies with respect to the average seasonal cycle. Carefully define the mathematical problem you would solve but do not give details on how you would solve it. Assume the average seasonal cycle is sinusoidal. [10 marks]

   (b) Explain how you would use your daily-average air temperature anomalies to check the assumption that the average seasonal cycle is sinusoidal. [10 marks]

2. Using a geophysical example of your choice, define the following terms: *data space*, *parameter space*, *null space*, *range*, *singular value*. For your example, explain, physically, the origin of the null-space. How is the singular value decomposition related to these quantities? (20 marks)

3. Draw matrix tableaux for the following problems:

   (a) The maximum-likelihood solution to an overdetermined least squares problem. [7 marks]

   (b) The singular value decomposition of a non-square matrix. [6 marks]

   (c) The solution to a tapered-weighted least squares problem. [7 marks]

4. Use the geometrical interpretation to explain the range of possible solutions for a quadratic interpolation problem. [20 marks]

5. (a) Define "tomography" using an example of your choice to explain the physical principles involved. [7 marks]

   (b) Explain why the inverse problem associated with tomography is under-determined. [5 marks]

   (c) Describe the nature of the general solution for a 2-dimensional tomography problem where the object being tested is square and is (conceptually) split into N different cells on each side. [8 marks]