270.307 Combining Measurements with Models
Final Test, Fall 2004

Name:

Answer ALL questions and attach this question sheet to the top of your answers. The expected number of marks is shown after each question. You may use your class notes and other sources to answer the test provided you cite all references. Do not discuss the test with anyone. Have a good holiday!

1. Consider the problem of finding $\mathbf{x}$ given $\mathbf{y}$, $\mathbf{E}$, and $\mathbf{y} = \mathbf{E}\mathbf{x}$ for square $\mathbf{E}$.
   (a) Explain how this problem in linear algebra can be viewed from a geometrical perspective when $\mathbf{E}$ is 2×2 or 3×3. (12 marks)
   (b) Describe the general solution for $\mathbf{x}$ when $\mathbf{E}$ is 3×3. (4 marks)
   (c) Describe the general solution for $\mathbf{x}$ when $\mathbf{E}$ is $N \times N$ (N is a positive integer). (4 marks)

2. Define and explain the following terms giving geophysical examples to illustrate each one: *variance*, *covariance*, *auto-covariance*, *covariance matrix*.
   (Note: the statistical meanings of these terms are required, not their meaning in probability theory). (20 marks)

3. (a) Explain how a dataset of scalar values recorded at discrete positions in space and times can be represented using empirical orthogonal functions (EOFs). Be sure to define and explain the terms: *empirical orthogonal function* and *principal value series*. (10 marks)

   (b) Assume the dataset contains noisy measurements from a system that is undergoing normal mode oscillations. What is the relationship between the empirical orthogonal functions, principal value series, and normal modes? In what sense can the dataset be filtered and compressed by a truncated EOF representation? (10 marks)

4. Using a geophysical example of your choice, define the following terms: *data space*, *parameter space*, *null space*, *range*, *singular value*. For your example, explain, physically, the origin of the null-space. How is the singular value decomposition related to these quantities? (20 marks)

5. (a) What is "tomography?" Give two distinct examples of tomography and explain the physical principles involved. (10 marks)

   (b) Compare and contrast the tapered-weighted least squares method with the singular-value decomposition method to solve a tomography problem. (10 marks)