Mathematics

The undergraduate program in the Department of Mathematics is intended for students interested in using mathematics to define and solve problems in the sciences, engineering, and other areas, as well as for those who wish to understand further the logical content, geometric meaning, and abstract reasoning of mathematics itself. A flexible program involving a broad selection of courses is a department tradition. The program begins by introducing students to the basics of algebra and mathematical analysis and then gives them the choice of exploring topics in theoretical mathematics or studying applications to physics, economics, engineering, computer science, probability, statistics, or mechanics.

The graduate program is designed primarily to prepare students for research and teaching in mathematics. It is naturally centered around the research areas of the faculty, which include algebraic geometry, algebraic number theory, differential geometry, partial differential equations, topology, several complex variables, algebraic groups, and representation theory. The program can be supplemented in applied directions by courses in theoretical physics, computer science, mechanics, probability and statistics offered in other departments of the Krieger School of Arts and Sciences and in the Department of Applied Mathematics in the Whiting School of Engineering.

The Faculty

Matthew Blair, Focused Research Grant Postdoctoral Fellow: harmonic analysis.

John M. Boardman, Professor: algebraic and differential topology.

Jason Brown, Director of Undergraduate Studies: dynamical systems, combinatorial group theory, symplectic topology.

Nero Budur, J. J. Sylvester Assistant Professor: algebraic geometry.

Michael Ching, J. J. Sylvester Assistant Professor: algebraic topology.

Caterina Consani, Associate Professor: algebraic/arithmetic geometry.

Daniel DeSilva, J. J. Sylvester Assistant Professor: analysis.

Carel Faber, Professor: algebraic and complex geometry.

Michael Goldberg, Assistant Professor: fourier analysis, PDEs, integral operators.

Philip Hartman, Emeritus Professor: differential equations, differential geometry.

Jun-ichi Igusa, Professor Emeritus: algebra, algebraic geometry, modular functions, number theory.

Jian Kong, Associate Research Scientist/Lecturer: algebraic geometry.

Chikako Mese, Associate Professor: geometric analysis.

Jean-Pierre Meyer, Professor Emeritus: algebraic topology, category theory.

William Minicozzi, Professor: differential geometry, partial differential equations, minimal surfaces.

Jack Morava, Professor: algebraic topology, mathematical physics.

Takashi Ono, Professor: algebra, number theory, algebraic groups.

Joseph A. Shalika, Professor: algebraic groups and representations, number theory.

Bernard Shiffman, Professor: several complex variables, differential geometry.

Vyacheslav V. Shokurov, Professor: algebraic geometry.

Christopher Sogge, Professor: Fourier analysis, partial differential equations.

Jian Song, J. J. Sylvester Assistant Professor: complex geometry.

Florin Spinu, Assistant Professor: analytic number theory.

Joel Spruck, Professor: partial differential equations, geometric analysis.

Richard Wentworth, Professor (Chair): mathematical physics, complex geometry.

W. Stephen Wilson, Professor: algebraic topology.

Steven Zelditch, Professor: quantum dynamics, spectral geometry, microlocal analysis.

Qiao Zhang, Assistant Professor: algebraic and complex geometry.

Steven Zucker, Professor: Hodge theory, algebraic geometry.

Joint Appointments

Jonathan A. Bagger, Professor (Physics and Astronomy): math/physics analysis.

Gregory Eyink, Professor (Applied Mathematics): pure and applied math.

Facilities

The university's Milton S. Eisenhower Library has an unusually extensive collection of mathematics literature, including all the major research journals. The stacks are open to students. The department also has a useful reference library, the Philip Hartman Library. Graduate students share departmental offices, and study space can also be reserved in the university library. Students may access the departments Unix and NT machines and the university's three mainframes from terminals in the department.

The Department of Mathematics is also home to the Japan-U.S. Mathematics Institute (JAMI), which each year invites six or more outstanding mathematicians from Japan. These visitors organize weekly seminars attended by graduate students and faculty.

Undergraduate Programs

Course Scheduling

Students usually begin by taking Calculus I-II, which is offered in three versions to meet the needs of students with different goals and interests. Students in mathematics, the physical sciences, and engineering are encouraged to begin with the 110.108-109 sequence or honors 110.113; students majoring in other subjects may wish to take the 110.106-107 sequence which relates the methods of calculus to the biological and social sciences. A one-term pre-calculus course 110.105 is offered for students who could benefit from additional preparation in the basic tools (algebra and trigonometry) used in calculus.

Entering students may receive course credit for Calculus I or Calculus I-II on the basis of the College Board AP exams. Students without AP credit should take a departmental placement exam to determine which course would be appropriate for them. For more information regarding placement, please visit: **www.math.jhu.edu**.

Linear Algebra (110.201), Calculus III (110.202), and Differential Equations (110.302) may be taken in any order after completing Calculus II (110.107 or 110.109). These courses are especially designed to acquaint students with mathematical methods relevant to engineering and the physical, biological, and social sciences. The department offers honors courses in both Linear Algebra and Calculus III (110.211-212). Additional courses oriented toward applications include 110.311, 110.417, 110.421 and 110.443. Students interested in the theoretical foundations of mathematics may select 110.401-402, 110.405-406, 110.413, and 110.439. Students planning to pursue further study in mathematics should work toward taking these theoretical courses as early as possible in their undergraduate years and are encouraged to take graduate-level courses as soon as they are qualified.

Requirements for the B.A. Degree:

In addition to the General Requirements for Departmental Majors, a candidate for the bachelor's degree in Mathematics is required to have credit for the courses listed below. All courses used to meet these requirements must be completed with a grade of C- or better. Honors Calculus III and Honors Linear Algebra (110.211-212) can be used in place of Calculus III and one course as described below.

- Calculus I, II, and III.
- 110.401 and one other term of algebra, either 110.204 or 110.402. (110.201 Linear Algebra does not satisfy this requirement.)
- 110.405 and one other term of analysis chosen from 110.311, 110.406, 110.417,
- 110.421, 110.439, and 110.443.
- Two terms of courses chosen from 110.201, 110.204, and mathematics courses at the 300-level or above. 110.211-212 Honors Calculus III and Linear Algebra may be used in place of Calculus III and one course in this requirement.
- Two terms in any one of the following areas of applications of mathematics, or other appropriate advanced courses as approved by the Director of Undergraduate Studies.
- Physics and Chemistry: 171.204, 301-302, 303-304, 312; 030.453.

  Probability and Statistics: 550.420, 426, 430, 620, 630.

  Applied Mathematics & Statistics: 550.361-362, 471, 472, 681.

  Economics: 180.301, 302, 311.

  Computer Science: 600.226, 435 and three credit courses numbered 605.425 and higher.

Requirements for a Minor in Mathematics:

Students with a major in another department may be awarded a minor in mathematics upon completion of satisfactory work in the following courses:

- Calculus I, II, and III.
- Four mathematics courses at the 200-level or above (excluding Calculus III), of which at least three are at the 300-level. A course in the Applied Mathematics and Statistics Department (at the corresponding level) may be substituted for one of the four courses.
- All courses used to meet these requirements must be completed with a grade of C- or better.

Honors Program in Mathematics:

As a general guideline, departmental honors are awarded to recipients of the B.A. degree who have completed Complex Analysis 110.311, as well as the Algebra sequence 110.401-402, the Real Analysis sequence 110.405-406, and one more course at the 400 level or above with at least a 3.6 average in these six courses.

J. J. Sylvester Prize

The J. J. Sylvester Prize in Mathematics, which carries a cash award, is given each year to an outstanding graduating senior majoring in mathematics.

The B.A./M.A. Program

By applying the same courses simultaneously toward the requirements for the B.A. and M.A. degrees, an advanced student can qualify for both degrees in four years. Admission to the program is by the standard graduate application form, which should be completed in the junior year. Students may contact the graduate program assistant for further information.

For candidates for the B.A./M.A. degree, at least a 3.0 average is required in the 400-level mathematics courses taken while resident at the university.

Graduate Programs

Admission

Admission to the Ph.D. program is based on academic records, letters of recommendation, and Graduate Record Examination scores. International applicants are required to submit a TOEFL score if English is not their native language.

Basic Program

Graduate study in mathematics is centered around 10 basic courses: 110.601-602, 110.605-606, 110.607-608, 110.611-612, 110.615-616, 110.617-618, 110.619-620, 110.631-632, 110.643-644, and 110.645-646. These courses are preliminary to research. The 700-level courses are designed to bring students abreast of recent developments and to prepare them for research in the area of their choice.

The basic graduate courses are built upon the foundations constituted by the 400-level courses 110.401-402, 110.405-406, and 110.413.

Requirements for the M.A. Degree:

- Completion, while resident at the university, of one basic graduate sequence, 110.601-602, 110.605-607, and demonstration of a deeper understanding by passing one written qualifying examination.
- A reading knowledge of French, German, or Russian, to be demonstrated by passing an examination given in the Department of Mathematics.

Requirements for the Ph.D. Degree:

- Satisfactory work in the analysis sequence 605 (Real Analysis I) and 607 (Complex Variables I) as well as the Algebra Sequence 601-602. Enrollment in these two sequences is mandatory for all first year graduate students unless they have received advanced placement by passing the corresponding written qualifying exam(s).
- Passing written qualifying exams by September of the second year in:
  1. Analysis (Real and Complex)
  2. Algebra
- Satisfactory work in at least three other 600-level courses in the second year of study.
- A reading knowledge of French, German, or Russian, to be demonstrated by passing an examination given in the Department of Mathematics.
- Passing an oral qualifying examination in the students chosen area of research. The topics on the exam are to be chosen in consultation with a faculty member (normally the student's future thesis adviser). This must be done by March of the third year.
- Some teaching of mathematics, usually at the undergraduate level, under the supervision of a faculty member.
- A written dissertation based upon independent research.
- The final Graduate Board oral examination, which is the dissertation defense.

Financial Aid

Most students admitted to the Ph.D. program receive teaching assistantships and full tuition fellowships. Exceptional applicants become candidates for one of the university's George E. Owen Fellowships, which carry no required duties the first year.

William Kelso Morrill Award

The William Kelso Morrill Award for excellence in the teaching of mathematics is awarded every spring to the graduate student who best exemplifies the traits of Kelso Morrill: a love of mathematics, a love of teaching, and a concern for students.

Undergraduate Courses

- *Please visit the Mathematics web page at www.math.jhu.edu for updated course offerings.*
- *Any course presented as a prerequisite must be completed with a grade of C- or higher*

110.105 (Q) Introduction to Calculus
This course starts from scratch and provides students with all the background necessary for the study of calculus. It includes a review of algebra, trigonometry, exponential and logarithmic functions, coordinates and graphs. Each of these tools will be introduced in its cultural and historical context. The concept of the rate of change of a function will be introduced. Not open to students who have studied calculus in high school.
4 credits

110.106-107 (Q) Calculus I, II (Biological and Social Sciences)
Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, introduction to differential equations, functions of several variables, linear systems, applications for systems of linear differential equations, probability distributions. (Many applications to the biological and social sciences will be discussed.)
4 credits

110.108-109 (Q) Calculus I, II (Physical Sciences and Engineering)
Differential and integral calculus. Includes analytic geometry, functions, limits, integrals and derivatives, polar coordinates, parametric equations, Taylor's theorem and applications, infinite sequences and series. (Some applications to the physical sciences and engineering will be discussed, and the courses are designed to meet the needs of students in these disciplines.)
4 credits

110.113 (Q) Honors Calculus II
This is an honors alternative to 107 or 109 and meets general requirement for Calculus 2. It is a more theoretical treatment of one variable integral calculus and is based on our modern understanding of the real number system as explained by Cantor, Dedekind, and Weierstrass. Students who want to know the "why's and how's" of Calculus will find this course rewarding. Students will be expected to already understand differential calculus (derivatives, differentiation, chain rule, optimization, related rates, etc), and will learn about the theory of integration, the fundamental theorem(s) of Calculus, applications of integration, and Taylor series. Prerequisite: A strong background in Calculus I, such as a 5 on the AP/BC Calculus exam, an "A" in 110.106or 110.108.
4 credits

110.201 (Q) Linear Algebra
Vector spaces, matrices, and linear transformations. Solutions of systems of linear equations. Eigenvalues, eigenvectors, and diagonalization of matrices. Applications to differential equations. Prerequisite: Calculus I.
4 credits

110.202 (Q) Calculus III (Calculus of Several Variables)
Calculus of functions of more than one variable: partial derivatives and applications; multiple integrals, line and surface integrals; Greens Theorem, Stokes Theorem, and Gauss Divergence Theorem. Prerequisite: Calculus II.
4 credits

110.204 (Q) Elementary Number Theory
The student is provided with many historical examples of topics each of which serves as an illustration of and provides a background for many years of current research in number theory. This course also provides the student with concrete examples of general abstract concepts studied in 110.401-402. Primes and prime factorization, congruences, Eulers function, quadratic reciprocity, primitive roots, solutions to polynomial congruences (Chevalleys theorem), Diophantine equations including the Pythagorean and Pell equations, Gaussian integers, Dirichlets theorem on primes. Prerequisite: Calculus I.
4 credits

110.211 (Q) Honors Calculus III
This course includes the material in Calculus III (202) with some additional applications and theory. Recommended for mathematically able students majoring in physical science, engineering, or especially mathematics. 211-212 used to be an integrated year-long course, but now the two are independent courses and can be taken in either order. Prerequisite: B+ or better in Calculus II, or 5 on the Calculus BC AP Exam.
4 credits

110.212 (Q) Honors Linear Algebra
This course includes the material in Linear Algebra (201) with some additional applications and theory. Recommended for mathematically able students majoring in physical science, engineering, or mathematics. 211-212 used to be an integrated year-long course, but now the two are independent courses and can be taken in either order. This course satisfies a requirement for the math major that its non-honors sibling does not. Prerequisite: Calculus II or III or equivalent, preferably honors.
4 credits

110.225 (Q) Putman Problem Solving
Problem solving course to prepare students for the Putman exam.
3 credits

110.228 (Q) Non-Euclidean Geometry
For 2,000 years, Euclidean geometry was the geometry. In the 19th century, new, equally consistent but very different geometries were discovered. This course will delve

into these geometries on an elementary but mathematically rigorous level. Prerequisite: high school geometry.
3 credits

110.302 (Q,E) Differential Equations with Applications
This is an applied course in ordinary differential equations, which is primarily for students in the biological, physical and social sciences, and engineering. The purpose of the course is to familiarize the student with the techniques of solving ordinary differential equations. The specific subjects to be covered include first order differential equations, second order linear differential equations, applications to electric circuits, oscillation of solutions, power series solutions, systems of linear differential equations, autonomous systems, Laplace transforms and linear differential equations, mathematical models (e.g., in the sciences or economics). Prerequisite: Calculus II.
4 credits

110.311 (Q) Methods of Complex Analysis
This course is an introduction to the theory of functions of one complex variable. Its emphasis is on techniques and applications, and it serves as a basis for more advanced courses. Functions of a complex variable and their derivatives; power series and Laurent expansions; Cauchy integral theorem and formula; calculus of residues and contour integrals; harmonic functions. Prerequisite: Calculus III.
4.5 credits

110.345 Basics Notions in Mathematics
This seminar course is intended to introduce majors and those interested in mathematics to a large collection of topics that they may not have seen before. It meets weekly with a different speaker each week. Prerequisites: Significant experience (at least two courses) with mathematics at the 200-level or above.
1 credit

110.401 (Q) Advanced Algebra I
An introduction to the basic notions of modern algebra. Elements of group theory: groups, subgroups, normal subgroups, quotients, homomorphisms. Generators and relations, free groups, products, commutative (Abelian) groups, finite groups. Groups acting on sets, the Sylow theorems. Definition and examples of rings and ideals. Introduction to field theory. Linear algebra over a field. Field extensions, constructible polygons, non-trisectability. Prerequisite: Linear Algebra.
4.5 credits

110.402 (Q) Advanced Algebra II
This is a continuation of 110.401. Theory of fields (continued). Splitting field of a polynomial, algebraic closure of a field. Galois theory: correspondence between subgroups and subfields. Solvability of polynomial equations by radicals. Modules over a ring. Principal ideal domains, structure of finitely generated modules over them. Applications.
4.5 credits

110.405 (Q) Analysis I
This course is designed to give a firm grounding in the basic tools of analysis. It is recommended as preparation (but may not be a prerequisite) for other advanced analysis courses. Real and complex number systems, topology of metric spaces, limits, continuity, infinite sequences and series, differentiation, Riemann-Stieltjes integration. Prerequisites: Calculus III, Linear Algebra.
4.5 credits

110.406 (Q) Analysis II
This course continues 110.405, with an emphasis on the fundamental notions of modern analysis. Sequences and series of functions, Fourier series, equicontinuity and the Arzela-Ascoli theorem, the Stone-Weierstrass theorem. Functions of several variables, the inverse and implicit function theorems, introduction to the Lebesgue integral. Prerequisite: 110.405.
4.5 credits

110.407-408 (Q,N) Geometry and Relativity
Special relativity: Lorentz transformation, Minkowski spacetime, mass, energy-momentum, stress-energy tensor, electrodynamics. Introduction to differential geometry: theory of surfaces, first and second fundamental forms, curvature, Gauss's theorema egregium, differentiable manifolds, connections and covariant differentiation, geodesics, differential forms, Stokes theorem. Gravitation as a geometric theory: Lorentz metrics, Riemann curvature tensor, tidal forces and geodesic deviation, gravitational redshift, Einstein eld equation, the Schwarzschild solution, perihelion precession, the deflection of light, black holes, cosmology. Prerequisites: Calculus II, Linear Algebra, General Physics II.
4.5 credits

110.409 (Q) Introduction to Algebraic Number Theory
This is an introduction to the arithmetic of rings of algebraic integers and more general Dedekind domains. It covers topics such as the unique factorization theorem for ideals in rings of algebraic integers, integral bases, the discriminant, the different, ramification, the finiteness theorem for ideal-class groups, Dirichlets theorem on groups of units of rings of algebraic integers, etc. Prerequisites: Algebra 110.401-402.
4 credits

110.411 (Q) Honors Complex Analysis
Study of functions of a complex variable, emphasis on interrelations with other parts of mathematics. Topics include Cauchy's theorems, singularities, gamma and zeta functions, elliptic functions, theta functions, Jacobi's triple product. Prerequisite: Calculus III, Linear Algebra.
4.5 credits

110.413 (Q) Introduction to Topology
The basic concepts of point-set topology: topological spaces, connectedness, compactness, quotient spaces, metric spaces, function spaces. An introduction to algebraic topology: covering spaces, the fundamental group, and other topics as time permits. Prerequisite: Calculus III.
4.5 credits

110.415 (Q) Honors Analysis I
This highly theoretical sequence in analysis is reserved for the most able students. The sequence covers the real number system, metric spaces, basic functional analysis, the Lebesgue integral, and other topics. Prerequisites: Calculus III and Linear Algebra.
4.5 credits

110.417 (Q,E) Partial Differential Equations for Applications
Characteristics, classification of second order equations, well-posed problems, separation of variables and expansions of solutions. The wave equation: Cauchy problem, Poissons solution, energy inequalities, domains of influence and dependence. Laplaces equation: Poissons formula, maximum principles, Greens functions, potential theory, Dirichlet and Neumann problems, eigenvalue problems. The heat equation: fundamental solutions, maximum principles. Prerequisites: Calculus III. Recommended: 110.405.
4.5 credits

110.423 (Q) Lie Groups for Undergraduates
This course is an introduction to Lie Groups and their representations at the upper undergraduate level. It will cover basic Lie groups such as SU (2), U (n), the Euclidean Motion Group and the Lorentz Group. This course is useful for students who want a working knowledge of group representations. We will also discuss some aspects of the role of symmetry groups in particle physics such as some of the formal aspects of the electroweak and the strong interactions. A good reference is the book Lie Algebras in Particle Physics by Howard Georgi. Prerequisite: Some group theory would be helpful.
4 credits

110.427 (Q) Introduction to the Calculus of Variations
The calculus of variations is concerned with finding optimal solutions (shapes, functions, etc.) where optimality is measured by minimizing a functional (usually an integral involving the unknown functions) possibly with constraints. In this introductory (self-contained) course, we will concern ourselves with one dimensional (often geometric) problems: brachistochrone, geodesics, minimum surface area of revolution, isoperimetric problem, curvature flows. We will run the course in a seminar style with active participation required. I will teach additional material as required (some differential geometry of curves and surfaces) to hold prerequisites to a minimum. Prerequisites: Calculus I, II, III.
4 credits

110.429 (Q) Mathematics of Quantum Mechanics
The basis of quantum mechanics is the Schrodinger equation. The focus of this course will be on one-dimensional Schrodinger equations. Topics include eigenvalue problems, bound states, scattering states, tunneling, uncertainty principle, dynamics, semi-classical limit. The ideas will be illustrated through many examples. Prerequisite: 110.302 (Differential Equations with Applications) or permission of the instructor.
4 credits

110.431 (Q) Introduction to Knot Theory
The theory of knots and links is a royal road to modern topology. The prerequisite for this course is a good grade in Calculus III, but the material will be mathematically sophisticated, and some familiarity with the notion of group would be helpful. We will start with braids, and work up to knots and links. The fundamental group of a knot or link complement will be the central algebraic focus, and spanning surfaces will be the main geometric tool. Together these lead very intuitively to homology groups (in low dimensions).
4 credits

110.439 (Q) Introduction to Differential Geometry
Theory of curves and surfaces in Euclidean space: Frenet equations, fundamental forms, curvatures of a surface, theorems of Gauss and Mainardi-Codazzi, curves on a surface; introduction to tensor analysis and Riemannian geometry; theorema egregium; elementary global theorems. Prerequisites: Calculus III, Linear Algebra.
4.5 credits

110.443 (Q,E) Fourier Analysis and Generalized Functions
An introduction to the Fourier transform and the construction of fundamental solutions of linear partial differential equations. Homogeneous distributions on the real line: the Dirac delta function, the Heaviside step function. Operations with distributions: convolution, differentiation, Fourier transform. Construction of fundamental solutions of the wave, heat, Laplace and Schrödinger equations. Singularities of fundamental solutions and their physical interpretations (e.g., wave fronts). Fourier analysis of singularities, oscillatory integrals, method of stationary phase. Prerequisites: Calculus III, Linear Algebra. Recommended: 110.405.
4.5 credits

110.462 (Q) Prime Numbers and Riemann's Zeta Function
This course is devoted to such questions as: How many prime numbers are there less than N? How are they spaced apart? Although prime numbers at first sight have nothing to do with complex numbers, the answers to these questions (due to Gauss, Riemann, Hadamard) involve complex analysis and in particular the Riemann zeta function. The best known unsolved conjecture in mathematics is about the zeros of Riemann zeta function, which control the distribution of primes. This course builds on 110.311 (Complex Analysis) and is an introduction to Analytic Number Theory for undergraduates. Prerequisite: 110.311.
4 credits

110.472 (Q) Differential Topology
Topics include manifolds, tangent spaces, immersions, submersions, transversality, intersection theory modulo 2, intersection numbers in the integers and Lefshetz fixed point theorem, and integration of differential forms on manifolds. Prerequisites: Calculus III and either 110.405 or 110.413.
4.5 credits

110.599 Independent Study, Undergraduate

Cross-Listed

171.204 Classical Mechanics
4 credits

Graduate Courses

110.601-602 Algebra
An introductory graduate course on fundamental topics in algebra to provide the student with the foundations for Number Theory, Algebraic Geometry, and other advanced courses. Topics include group theory, commutative algebra, Noetherian rings, local rings, modules, rudiments of category theory, homological algebra, field theory, Galois theory, and non-commutative algebras. Prerequisites: 110.401-402.

110.604 Strategies for Computer-Assisted Mathematics Instruction
This course is designed to introduce teaching assistants to the Maple program and to explore strategies using Maple in the teaching of undergraduate mathematics. It may be required as a part of their normal duties. Others may enroll only with permission of instructor.

110.605-606 Real Variables
Measure and integration on abstract and locally compact spaces (extension of measures, decompositions of measures, product measures, the Lebesgue integral, differentiation, LP-spaces); introduction to functional analysis; integration on groups; Fourier transforms. Prerequisites: 110.405, 110.413, or equivalent.

110.607-608 Complex Variables
Analytic functions of one complex variable. Topics include Mittag-Leffler Theorem, Weierstrass factorization theorem, elliptic functions, Riemann surfaces, divisors and line bundles, Riemann-Roch theorem, Picard theorem, and Nevanlinna theory. Prerequisites: 110.311 and 110.405.

110.611-612 Several Complex Variables
Domains of holomorphy and pseudoconvexity, Levi pseudoconvexity. The Weierstrass preparation and division theorems, properties of the local ring of germs of holomorphic functions, complex analytic varieties, the Ruckert Nullstellensatz. Sheaves and cohomology, coherent analytic sheaves, Okas coherence theorem, Dolbeault cohomology. Additional topics such as Chows theorem, L2 cohomology, integral formulas, Cartans Theorems A and B, compact complex manifolds. Prerequisite: 110.413. Recommended: 110.605-606.

110.615-616 Algebraic Topology
Polyhedra, simplicial and singular homology theory, Lefschetz fixed-point theorem, cohomology and products, homological algebra, Künneth and universal coefficient theorems, Poincaré and Alexander duality theorems. Prerequisites: 110.401, 110.413.

110.617-618 Number Theory
Topics in advanced algebra and number theory, including local fields and adeles, Iwasawa-Tate theory of zeta-functions and connections with Hecke's treatment, semi-simple algebras over local and number fields, adeles geometry. Prerequisites: 110.401-402.

110.619-620 Lie Groups and Lie Algebras
Lie groups and Lie algebras, classification of complex semi-simple Lie algebras, compact forms, representations and Weyl formulas, symmetric Riemannian spaces. Prerequisite: 110.402.

110.631-632 Partial Differential Equations
An introductory graduate course in partial differential equations. Classical topics include first order equations and characteristics, the Cauchy-Kowalevski theorem, Laplaces equation, heat equation, wave equation, fundamental solutions, weak solutions, Sobolev spaces, maximum principles. The second term focuses on special topics such as second order elliptic theory. Prerequisites: 110.605-606.

110.635-636 Microlocal Analysis
Microlocal analysis is the geometric study of singularities of solutions of partial differential equations. The course will begin by introducing the geometric theory of (Schwartz) distributions: Fourier transform and Sobolev spaces, pseudo-differential operators, wave front set of a distribution, elliptic operators, Lagrangean distributions, oscillatory integrals, method of stationary phase, Fourier integral operators. The second semester will develop the theory and apply it to special topics such as asymptotics of eigenvalues/eigenfunctions of the Laplace operator on a Riemann manifold, linear and non-linear wave equation asymptotics of quantum systems, Bochner-Riesz means, maximal theorems. Prerequisites: 110.605-606. Recommended: 110.631.

110.641 Harmonic Analysis
Harmonic analysis begins with Fourier analysis on Euclidean space. Topics include Littlewood-Paley theory, oscillatory integrals, restriction theorems for the Fourier transform, Bochner-Riesz means and multiplier theorems.

110.643-644 Algebraic Geometry
Afne varieties and commutative algebra. Hilberts theorems about polynomials in several variables with their connections to geometry. General varieties and projective geometry. Dimension theory and smooth varieties. Sheaf theory and cohomology. Applications of sheaves to geometry; e.g., the Riemann-Roch Theorem. Other topics may include Jacobian varieties, resolution of singularities, geometry on surfaces, connections with complex analytic geometry and topology, schemes. Prerequisites: 110.601-602.

110.645-646 Riemannian Geometry
Differential manifolds, vector fields, flows, Frobenius' theorem. Differential forms, deRham's theorem, vector bundles, connections, curvature, Chern classes, Cartan structure equations. Riemannian manifolds, Bianchi identities, geodesics, exponential maps. Geometry of submanifolds, hypersurfaces in Euclidean space. Other topics as time permits, e.g., harmonic forms and Hodges theorem, Jacobi equation, variation of arc length and area, Chern-Gauss-Bonnet theorems. Prerequisites: 110.405, 110.413. 110.660 Qualifying Exam Problems.

110.705 Random Matrices
This course will provide a broad introduction to the subject of random matrix theory and some of its applications. After a review of the classical theory of limiting statistics of eigenvalues for the Gaussian and Circular ensembles, we shall focus on orthogonal and skew-orthogonal polynomial methods to study limiting statistics for more general matrix ensembles. Recent results toward proving statistical universal properties of the eigenvalues will also be covered, and applications including random permutations and random growth models will be discussed. A background in analysis at the undergraduate level is required.

110.711-712 Quantum Cohomology
Quantum cohomology is the currently fashionable name for a way of organizing recent new insights into a large class of nonlinear global problems in the calculus of variations. Work of Gromov, Floer, Witten, and others on an infinite-dimensional version of Morse theory leads to new kinds of invariants for geometrical structures, with applications including the enumeration of rational curves on classical algebraic varieties, new invariants of four-manifolds, proof of Arnolds conjecture on stationary points of dynamical systems, as well as to questions motivated by quantum field theory. Although the results are formulated in the language of algebraic topology, the theory of elliptic differential equations is the main technical tool. Prerequisites: some knowledge of ordinary cohomology and some familiarity with either PDE or differential geometry.

110.721-722 Homotopy Theory
Homotopy groups, fiber spaces, fiber bundles, Hurewicz isomorphism theorem, local coefficients, spectral sequences, cohomology operations, obstruction theory, Postnikov systems. Prerequisites: 110.615-616.

110.723-724 Topics in Automorphic Functions

110.725-726 Topics in Analysis

110.727-728 Topics in Algebraic Topology

110.729-730 Topics in Several Complex Variables

110.733-734 Topics in Algebraic Number Theory

110.735-736 Topics in Hodge Theory

110.737-738 Topics in Algebraic Geometry

110.739 Topics in Analytic Number Theory

110.741-742 Topics in Partial Differential Equations

110.751-752 Topics in Group Representations

110.799 Thesis Research

110.800 Independent Study, Graduate