

MATERIAL TESTING

Focus:

In this lab, we are interested in the strength of uncooked spaghetti. For each of these materials you will measure three qualities: **buckling load, yield or fracture strength, and Young's modulus.**

Part 1: Buckling load**Overview:**

If you try to compress a cylindrical rod by applying an increasing force against its end, you will reach a point where the rod no longer remains straight, but begins to buckle (bend). The force at which this happens is called the Euler buckling load P . Here we're interested in deducing an expression for $P = f(L, R) = AE\pi^2/(2kL/R)^2$, where L is the length of the rod and R is the radius, and k is a constant which depends on how the rod is held. If the rod is held in such a way to allow free rotation at the ends, then $k=1$ and $P = AE\pi^2/(2L/R)^2$.

Many physical phenomena are governed by single expression "power laws". In this case we might expect $P = \beta L^m R^n$, where m and n are most often integers (but sometimes halves or thirds), and β is a constant that takes into account other fixed parameters. For this problem $\beta = E\pi^3/(4k^2)$, where E is Young's modulus and k is a constant which depends on how the rod is held. If the rod is held in such a way to allow free rotation at the ends, then $k = 1$. Then $\beta = E\pi^3/4$.

Young's modulus of elasticity E is a measure of the inherent rigidity of a material. For a given geometric configuration, a material with larger E deforms less under the same stress. (Stress σ is load per unit area; strain ϵ is deformation per unit length.) Most materials are "elastic" (at least through some range of strains), i.e., stress is proportional to strain. In these elastic regions $E = \sigma/\epsilon = \text{stress/strain}$.

Procedure:

We want to develop this formula by experiment and establish a value of E for spaghetti. To do this, measure the buckling load by pushing on the upper end of a length of material whose lower end is supported by a postal scale. (Don't grip the material at the end, because you want it to be free to pivot.)

For each of the different diameters of spaghetti (you may have three or four) record the buckling load vs. length of at least twenty-five specimens. Use lengths which will give a good distribution of buckling loads up to about 1.0Kg (force) for the thicker stuff, and to about 0.2Kg (force) for the thinnest stuff. Make sure you know what the spaghetti diameters are. (Use the linen tester or caliper [or other means] to obtain the diameters.)

On a single graph, plot P vs. L for all the spaghetti diameters. Of course, label each set of data with different symbols. What do you get? Collectively, a mess; but within data sets relatively smooth curves. These are the raw results. The object now is to find an expression relating P to L and R , i.e. to find m and n . How best to do that? First consider data from the spaghetti. If R is constant, then the proposed equation is reduced to $P = \gamma L^m$. Figure out how to plot your data to deduce m . Similarly fix L and deduce n from the equation $P = \lambda R^n$. [Hint: think logarithms.]

Now you should be able to back out E for the different diameters of spaghetti. For each of the spaghetti diameters, calculate the average E based on all the observations. Is E different for the four diameters? (They should be approximately the same.) Take the average value of your four E 's and call that the "best" value. Next, produce a scatter plot of P/R^n vs. L for all your data (i.e., data from all four diameters) and superpose the analytic curve of P/R^n vs. L using your "best" value for E . (In principle, E should not depend on L . Does it?) Make sure your plots are adequately labeled. Do all the data collapse onto the curve? Should they? Why? Finally, it should be possible to collapse all your data onto a single curve by plotting $P/(ER^n)$ vs. L . Do it.

Write-up:

Produce a brief report, including well-labeled graphs, which describes the experiment, the method, the results, and the errors.

Part 2: Yield strength, fracture strength

Overview:

If you pull on both ends of a rod with an increasing amount of force (making sure to apply the force only along the length of the rod), the rod will deform plastically or irreversibly at some point. The force will have exceeded the material's yield strength. Of course, depending on the material, the rod may not simply snap in two. It might experience deformation in the form of necking (a gradual reduction in cross-sectional area) before it breaks. Under tensile loading, brittle materials tend to fracture without deformation, while ductile materials typically experience plastic deformation and necking before breaking.

The force required to pull a rod apart is dependent on two things: the inherent strength of the material, and the cross-sectional area of the rod. The larger the cross-section, the larger the force required to pull it apart. This makes sense because a larger cross-sectional area means that there are more atomic bonds to pull apart along a line of fracture, which in turn necessitates a larger force. In order to create some consistency in reported values, strength is given in terms of force/area with units p.s.i. (pounds per square inch) or Pascals (Newtons per square meter). Thus, a material should have a constant strength regardless of its cross-sectional area. However, imperfections and non-uniformities in a material can cause deviations in the measured values of strength.

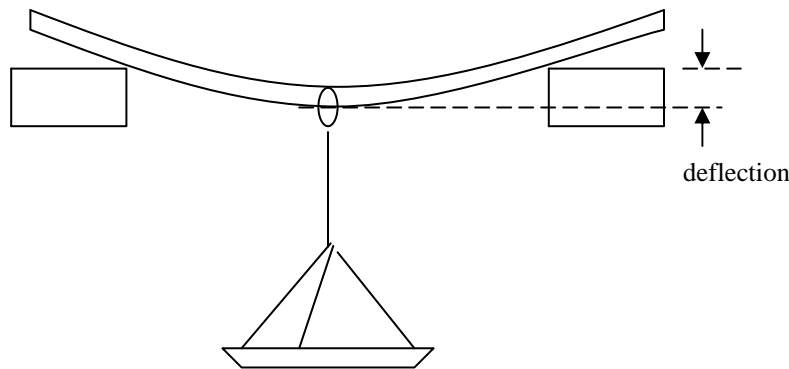
Usually strength is measured by pulling on a specimen until it breaks. But there are also other ways of determining strength, one of which is to apply a bending force on a specimen until it breaks. Both techniques will be used in the following two experiments.

Procedure 1 (indirect measure of strength):

Here you want to measure the deflection of spaghetti under a bending load. The experimental procedure is simple: lay a piece of material between two supports, hang a weight mid-way between its support points, and measure the material's deflection. Then infer Young's modulus and the ultimate yield strength or fracture strength of the material.

Strength is related to bending in the following way: When a specimen is bent its material is being strained (stretched or compressed). Atoms are pushed closer together on the inside of the bend and pulled farther apart on the outside of the bend. Since these atoms are no longer at their equilibrium distance, they exert reactionary forces within the specimen which balance the bending load. Up to a point. At some degree of bending, the specimen yields or fractures. That is, it irreversibly deforms. For brittle materials (spaghetti), the specimen may simply fracture. For non-brittle materials (e.g., aluminum), the specimen yields and takes on a permanent deformation. These points are called the fracture strength and yield strength, respectively. If the material breaks, the atoms on the outside of the bend have been pulled too far apart to hold the material together. That is, the material has reached its maximum strain ϵ_{\max} .

The apparatus for this experiment consists of a support frame, a loading platform, and a handful of lead weights. Determine the weight of the loading platform and the weights.



For each of the diameters of spaghetti, perform a load vs. deflection measurement on at least two samples. First, insert the specimen through the eye of the swivel on the loading platform. Then lay the specimen across the two grooved supports. These grooves allow for three different support widths. Experiment to see which is most suitable for your specimen. Make sure that the bending load is placed midway between the supports. Measure the deflection with the empty loading platform. Then, begin to add weights, recording the deflection with each new addition. At some maximum deflection, the specimen will yield. That's a value you're especially interested in.

The relationship between deflection y and the bending load P is $P = 48yIE/L^3$, where E is Young's modulus, L is the length between the supports, and I is the cross-sectional moment of inertia of the specimen. For a circular cross section, $I = \frac{1}{4}\pi R^4$. Plot P as a function of y for each of the four diameters of spaghetti—all on the same graph, if possible. E is related to the slope of the line formed by the data. Is it a line? What sort of trends do you see? Calculate E based on the best line through the data.

Now for an estimate of the fracture or yield strength. Under a bending load, a specimen will tend to distribute the stress uniformly between the support points, i.e., the specimen will take the form of a circular arc. A little trigonometry will show that the radius of that

arc is $\rho = \frac{1}{2y} \left[\left(\frac{L}{2} \right)^2 + y^2 \right]$. As the specimen is bent, strain is imposed on the material—

from maximum compressive strain on the inside of the arc to maximum tensile strain on the outside of the arc. At the center of the material there is a neutral plane where the strain passes from compression to tension. We can deduce strain ϵ in the material as simply

$\epsilon = C/\rho$, where C is the distance from the neutral plane. To calculate ϵ_{\max} , we need the value of ρ at which the material fractured or yielded and the maximum value of C for the material. In this case C is simply R , the radius of our specimen. $\epsilon_{\max} = R/\rho$.

We need one final step—to convert maximum strain to maximum stress, i.e. fracture or yield strength. Use Hook's Law $\sigma_{\max} = E \epsilon_{\max}$. σ_{\max} is the maximum stress that you can

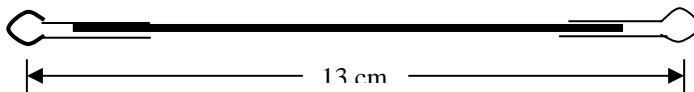
expect out of the material. Recall that stress is force per unit area. Carry out these calculations for each of your six specimens. From these calculations, deduce the fracture strength of the four different diameters of spaghetti. Are the experimental strengths indeed proportional to their cross-sectional areas? Explain.

Procedure 2 (direct measurement of tensile strength):

Here you want to directly measure the fracture strength of spaghetti using a lever-and-fulcrum apparatus. A principal problem in conducting this experiment, however, is how to hold the specimen, so that the specimen breaks as a natural consequence of its material properties and not because of your imposing additional stresses associated with holding the ends.

One solution is to epoxy cotter pins to both ends of each sample of spaghetti so that the specimen can be held by pins passing through the heads of the cotter pins. **Specimens must be prepared at least four hours before the beginning of this lab.** First, pry the prongs of the cotter pins apart so that a uniform gap is created which is wide enough to accommodate the spaghetti. Next, make two dots 13cm apart on a piece of paper and slip this paper under a piece of wax paper. The dots on the paper will serve as a guide for making correct-length specimens; the wax paper will keep the epoxy from sticking to the paper. Place two cotter pins so that the centers of their heads are on the dots and their prongs are pointing toward one another. Take a piece of spaghetti approximately 12cm long and place it between the prongs of the two cotter pins. Epoxy the spaghetti to the prongs. Don't let the spaghetti or the epoxy extend into the heads of the cotter pins. And make sure that the centers of the heads of the cotter pins are separated by $13\text{cm} \pm 1\text{mm}$ --the preferred separation for the load-measuring device. Although you will be using "5-minute" epoxy, adequate strength is obtained only after several hours of curing. Prepare 3 specimens each of the different diameters of spaghetti.

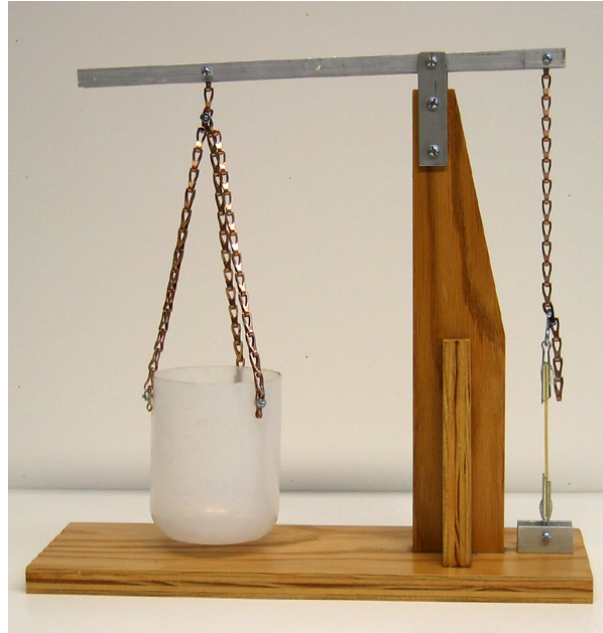
Your specimens should look like this:



The whole apparatus--including a specimen ready for testing-- is shown below.

To test the tensile strength of a specimen, first pin the lower end of the specimen (cotter pin) to the bracket on the base using a pin (or screw). Then attach the upper end of the specimen to the chain via an "S-hook" at a position to make the lever arm essentially horizontal. (right-hand side of the figure). Attach the bottle to one of three positions on the other side of the lever using a pin (or screw). Which position depends on the specimen: with thinner spaghetti, use the position closest to the fulcrum; with thicker spaghetti, use one of the farther positions. The bottle should hang a few centimeters off the base. If not, change the position of the S-hook attached to the specimen. Load the

bottle until the specimen fractures. Record that load. Don't forget that the lever may be providing a mechanical advantage!



Write-up:

Generate a plot of measured fracture strength vs. cross-sectional area for each of the two methods—the indirect and the direct. Normalized, here, means tensile strength divided by cross-sectional area. (This presumes that strength is proportional to cross-sectional area—not a bad presumption.) The plot you should obtain should be roughly uniform across the four diameters (with scatter, or course). Write a report which discusses the experimental technique and explain why there is a fairly wide distribution of values. Where does this variation come from? Are the thin vs. thick material strengths consistent with cross-sectional area? Do the indirect and direct measurements of fracture strength yield similar values. If not, explain. How would you use these data to obtain a "best guess" of fracture strength/area? Note: your experimental errors are not purely random; they are biased.