

The Logarithmic Function:

$$L = \log_b N$$

The inverse function is: $N = b^L$

For example:

$$\log_2 8 = 3 \quad \text{since } 8 = 2^3$$

$$\log_{10} 0.01 = -2 \quad \text{since } 0.01 = 10^{-2}$$

$$\log_5 5 = 1 \quad \text{since } 5 = 5^1$$

$$\log_b 1 = 0 \quad \text{since } 1 = b^0$$

Selected Algebra Topics

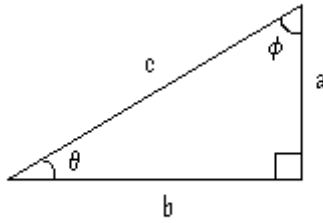
Basic Laws of Exponents

Law	Example
$a^m a^n = a^{m+n}$	$x^5 x^{-2} = x^3$
$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{x^5}{x^3} = x^2$
$(a^m)^n = a^{mn}$	$(x^{-2})^3 = x^{-6}$
$(ab)^m = a^m b^m$	$(xy)^2 = x^2 y^2$
$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{x}{y}\right)^2 = \frac{x^2}{y^2}$
$a^{-m} = \frac{1}{a^m}, a \neq 0$	$x^{-3} = \frac{1}{x^3}$
$a^0 = 1, a \neq 0$	$2(3x)^0 = 2(1) = 2$
$a^1 = a$	$(3x^2)^1 = 3x^2$

Laws for fractional exponents

Law	Example
$a^{m/n} = \sqrt[n]{a^m}$	$x^{2/3} = \sqrt[3]{x^2}$
$\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}, b \neq 0$	$\frac{\sqrt[3]{16}}{\sqrt[3]{2}} = \sqrt[3]{8} = 2$
$a^{1/2} = \sqrt[2]{a^1} = \sqrt{a}, a \geq 0$	$\sqrt{25} = 5, (\text{not } \pm 5)$

Trigonometric Identities



$\sin(\theta) = \frac{a}{c}$	$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{c}{a}$
$\cos(\theta) = \frac{b}{c}$	$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{c}{b}$
$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)} = \frac{a}{b}$	$\cot(\theta) = \frac{\cos(\theta)}{\sin(\theta)} = \frac{b}{a}$

$$\sin(-x) = -\sin(x)$$

$$\csc(-x) = -\csc(x)$$

$$\cos(-x) = \cos(x)$$

$$\sec(-x) = \sec(x)$$

$$\tan(-x) = -\tan(x)$$

$$\cot(-x) = -\cot(x)$$

$$\sin^2(x) + \cos^2(x) = 1$$

$$\tan^2(x) + 1 = \sec^2(x)$$

$$\cot^2(x) + 1 = \csc^2(x)$$

$$\sin(x \pm y) = \sin(x)\cos(y) \pm \cos(x)\sin(y)$$

$$\cos(x \pm y) = \cos(x)\cos(y) \pm \sin(x)\sin(y)$$

$$\tan(x \pm y) = \frac{\tan(x) \pm \tan(y)}{1 \pm \tan(x)\tan(y)}$$

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$$

$$\tan(2x) = \frac{2\tan(x)}{1 - \tan^2(x)}$$

$$\sin^2(x) = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cos^2(x) = \frac{1}{2} + \frac{1}{2} \cos(2x)$$

$$\sin(x) - \sin(y) = 2 \sin\left(\frac{(x-y)}{2}\right) \cos\left(\frac{(x+y)}{2}\right)$$

$$\cos(x) - \cos(y) = -2 \sin\left(\frac{(x-y)}{2}\right) \sin\left(\frac{(x+y)}{2}\right)$$

Given Triangle abc, with angles A,B,C; a is opposite to A, b is opposite to B, and c is opposite to C:

$$\text{Law of Sines: } \frac{a}{\sin(A)} = \frac{b}{\sin(B)} = \frac{c}{\sin(C)}$$

Law of Cosines:

$$c^2 = a^2 + b^2 - 2ab \cos(C)$$

$$b^2 = a^2 + c^2 - 2ac \cos(B)$$

$$a^2 = b^2 + c^2 - 2bc \cos(A)$$

$$\text{Law of Tangents: } \frac{(a-b)}{(a+b)} = \frac{\tan\left(\frac{1}{2}(A-B)\right)}{\tan\left(\frac{1}{2}(A+B)\right)}$$

Important Statistics Formulas:

Parameters:

$$\text{Population mean: } \mu = \frac{(\sum X_i)}{N}$$

$$\text{Population Standard Deviation: } \sigma = \sqrt{\frac{\sum (X_i - \mu)^2}{N}}$$

$$\text{Population Variance: } \sigma^2 = \frac{\sum (X_i - \mu)^2}{N}$$

$$\text{Standardized Score: } Z = \frac{(X - \mu)}{\sigma}$$

$$\text{Population Correlation Coefficient: } \rho = \left[\frac{1}{N} \right] * \Sigma \left\{ \left[\frac{(X_i - \mu_x)}{\sigma_x} \right] * \left[\frac{(Y_i - \mu_y)}{\sigma_y} \right] \right\}$$

Statistics:

$$\text{Sample mean: } \bar{x} = \frac{(\sum x_i)}{n}$$

$$\text{Sample standard deviation: } s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{(n-1)}}$$

Sample variance: $s^2 = \frac{\Sigma(x_i - \bar{x})^2}{(n-1)}$

Sample Correlation coefficient: $r = \left[\frac{1}{(n-1)} \right] * \Sigma \left\{ \left[\frac{(x_i - \bar{x})}{s_x} \right] * \left[\frac{(y_i - \bar{y})}{s_y} \right] \right\}$

Normal Distribution Formula: $\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$

Or $\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$

Simple Linear Regression:

Simple linear regression line: $\hat{y} = b_0 + b_1x$

Regression coefficient: $b_1 = \frac{\Sigma[(x_i - \bar{x})(y_i - \bar{y})]}{\Sigma(x_i - \bar{x})^2}$

Regression slope intercept: $b_0 = \bar{y} - b_1 * \bar{x}$

Standard error of regression slope: $s_{b_1} = \frac{\sqrt{\frac{\Sigma(y_i - \hat{y}_i)^2}{(n-2)}}}{\sqrt{\Sigma(x_i - \bar{x})^2}}$

Random Variables:

Expected value of X: $E(X) = \mu_x = \Sigma[x_i * P(x_i)]$

Variance of X: $Var(X) = \sigma^2 = \Sigma[x_i - E(x)]^2 * P(x_i) = \Sigma[x_i - \mu_x]^2 * P(x_i)$

Normal Random Variable: $z - score = z = \frac{(x - \mu)}{\sigma}$

Expected value of sum of random variables: $E(X + Y) = E(X) + E(Y)$

Expected value of difference between random variables: $E(X - Y) = E(X) - E(Y)$

Variance of the sum of *independent* random variables:

$$Var(X + Y) = Var(X) + Var(Y)$$

Variance of the difference between *independent* random variables:

$$Var(X - Y) = Var(X) - Var(Y)$$

Sampling Distributions:

Standard deviation of the mean: $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

Standard Error:

Standard error of the mean: $SE_{\bar{x}} = s_{\bar{x}} = \frac{s}{\sqrt{n}}$

Taylor series expansion:

$$f(x + \Delta x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(x)\Delta x^n}{n!} = f(x) + f'(x)\Delta x + \frac{f''(x)\Delta x^2}{2!} + \frac{f'''(x)\Delta x^3}{3!} + \dots$$

Maclaurin series expansion: $f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(0)x^n}{n!}$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} + \dots$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} + \dots$$