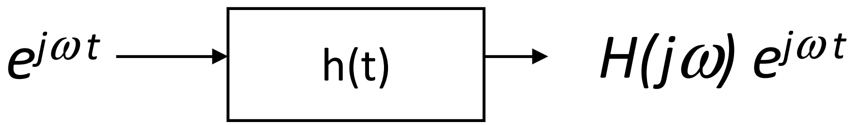
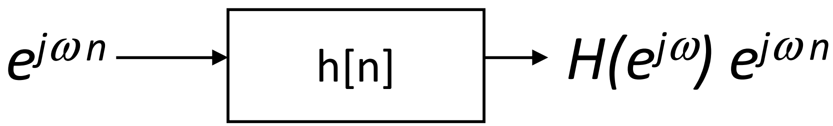
Complex exponentials as input to LTI systems



$$H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$



$$H(e^{j\omega}) = \sum_{n=-\infty}^{\infty} h[n] e^{-j\omega n}$$

Cos as input… use Euler formula

LCC Differential equation

- 1^{st} order: $y'(t) + a\, y(t) = x(t)$

- Homogenous solution/ zero input / natural response:

  $y'_h(t) = -ay_h(t)$

  $y_h(t) = e^{-at}$
- Solution to x(t) = δ(t), with zero initial condition

  $y(t) = h(t) = e^{-at}\, u(t)$
- What is initial rest condition:

  $x(t) = 0$, for $t <= t_0$

  $y(t) = 0$, for $t <= t_0$
- If zero initial condition (initially at rest) is imposed: equation is describing LTI and causal system.

LCC Difference equation

- 1^{st} order: $y[n] - a\, y[n-1] = x[n]$

- Homogenous solution/ zero input / natural response:

  $y_h[n] = a y_h[n-1]$

  $y_h[n] = a^n$
- Solution to $x[n] = \delta[n]$, with zero initial condition

  $y[n] = h[n] = a^n u[n]$
- Use initial rest condition to find particular solution:

  $x[n] = 0$, *for* $n <= n_0$

  $y[n] = 0$, *for* $n <= n_0$
- If zero initial condition (initially at rest) is imposed: equation is describing LTI and causal system.

LCC Differential equation

- N^{th} order: $$\sum_{k=0}^{N} a_k y^{(k)}(t) = \sum_{k=0}^{M} b_k x^{(k)}(t)$$
- 1^{st} order: $y'(t) + a\, y(t) = b\, x(t)$

- Homogenous solution/ zero input / natural response:
  $y'_h(t) = -ay_h(t)$
  $y_h(t) = e^{-at}$
- Solution to x(t) = δ(t), with zero initial condition:
  $y(t) = h(t) = b\, e^{-at}\, u(t)$
  $\delta(t)$ *is changing the output instantaneously.*
- Zero initial condition:
  $x(t) = 0$, *for* $t <= t_0$
  $y(t) = 0$, *for* $t <= t_0$
- If zero initial condition (initially at rest) is imposed: equation is describing LTI and causal system.

LCC Difference equation

- N^{th} order: $$\sum_{k=0}^{N} a_k y[n-k] = \sum_{k=0}^{M} b_k x[n-k]$$
- 1^{st} order: $y[n] - a\, y[n-1] = x[n]$

- Homogenous solution/ zero input / natural response:

  $y_h[n] = a y_h[n-1]$

  $y_h[n] = a^n$
- Solution to $x[n] = \delta[n]$, with zero initial condition

  $y[n] = h[n] = a^n u[n]$
- Initial rest condition:

  $x[n] = 0$, for $n <= n_0$

  $y[n] = 0$, for $n <= n_0$
- If zero initial condition (initially at rest) is imposed: equation is describing LTI and causal system.