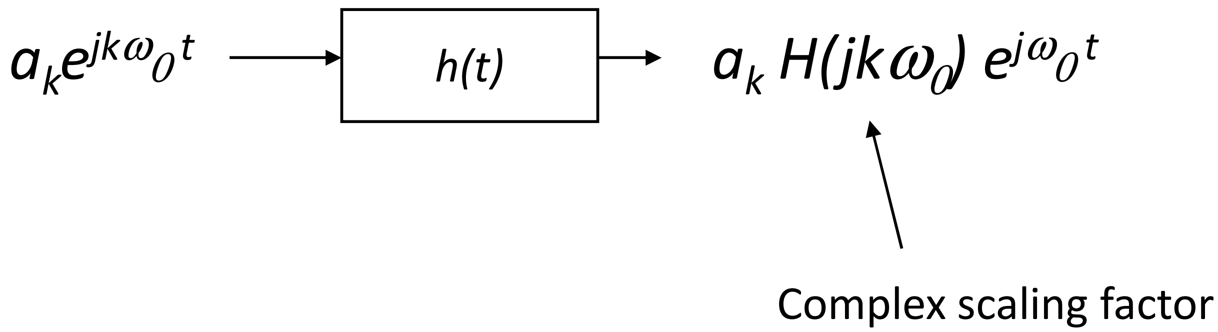
Basic signals

- Why use complex exponentials?
  - Because they are useful building blocks which can be used to represent large and useful classes of signals
  - Response of LTI systems to these basic signals is particularly simple and useful.



Fourier Series Representation of CT Periodic Signals

x(t) = x(t+T) for all t

- Smallest such T is the *fundamental period*
- $\omega_0 = 2\pi/T$ is the *fundamental frequency*

$e^{j\omega t}$ periodic with period T $\Leftrightarrow$ $\omega = k\omega_0$

$$\Downarrow$$

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{j2\pi kt/T}$$

- Periodic with period T
- $\{a_k\}$ are the *Fourier (series) coefficients*
- k = 0 DC
- k = ±1 first harmonic
- k = ±2 second harmonic

CT Fourier Series Pair

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jkw_0 t}$$ (Synthesis Equation)

$$a_k = \frac{1}{T}\int_T x(t) e^{-jkw_0 t} dt$$ (Analysis Equation)

Examples

- $x(t) = \cos(4\pi t) + 2\sin(8\pi t)$
- Periodic square wave (from lecture)
- Periodic impulse train

$$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$$

Even and Odd Functions

- Even functions are defined by the property:

$$f(x) = f(-x)$$

- Odd functions are defined by the property:

$$f(x) = -f(-x)$$

Some notes on CT Fourier series

- For k = 0, $a_0 = s_T\, x(t)\, dt$ ← mean value over the period (DC term)
- If $x(t)$ is real, $a_k = a^*_{-k}$
- If $x(t)$ is even, $a_k = a_{-k}$
- If $x(t)$ is odd, $a_k = -a_{-k}$

- If $x(t)$ is real and even, $a_k = a^*_{-k} = a_{-k}$ → a_k*'s will only have real terms*
- If $x(t)$ is real and odd, $a_k = a^*_{-k} = -a_{-k}$ → a_k*'s will only have odd terms*

The CT Fourier Transform Pair

$$x(t) \leftrightarrow X(j\omega)$$

$$X(j\omega) = \int_{-\infty}^{+\infty} x(t)e^{-j\omega t}dt$$

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega)e^{j\omega t}d\omega$$