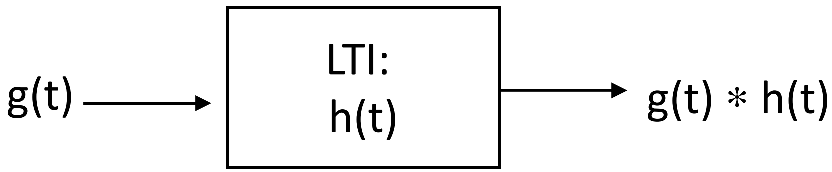
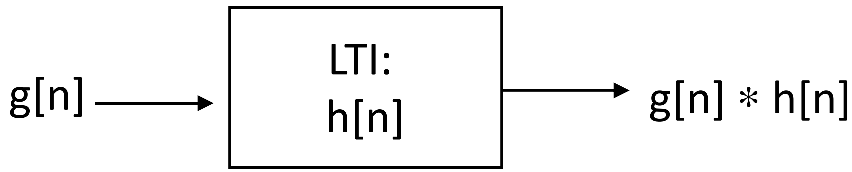
# Convolution

$$g(t) * h(t) = \int_{-\infty}^{\infty} g(\tau)h(t-\tau)d\tau$$

$$g[n] * h[n] = \sum_{k=-\infty}^{\infty} g[k]h[n-k]$$




Example: $g[n] = u[n] – u[3-n]$

$h[n] = \delta[n] + \delta[n-1]$

# Convolution methods:

- Method 1: “running sum”
  1. Plot $x$ and $h$ vs. $m$
  2. Flip h over vertical axis to get $h[-m]$
  3. Shift $h[-m]$ to obtain $h[n-m]$
  4. Multiply to obtain $x[m]h[n-m]$
  5. Sum on $m$, $\sum_m x[m]\, h[n-m]$
  6. Increment n and repeat steps 3~6

# Convolution methods:

- **Method 2: Superposition method**
  - We know $\delta[n] \rightarrow h[n]$ and the system is LTI.
  - Therefore, $a_0\delta[n-n_0] \rightarrow a_0h[n-n_0]$
  - $a_0\delta[n-n_0] + a_1\delta[n-n_1] + \ldots \rightarrow a_0h[n-n_0] + a_1h[n-n_1] \ldots$
  - Since the input can be expressed as a sum of shifted unit samples:

    $x[n] = \ldots x[-2]\delta[n+2] + x[-1]\delta[n+1] + x[0]\delta[n] + x[1]\delta[n-1] + \ldots$
  - The out is then a sum of shifted unit sample responses:

    $x[n_0]\delta[n-n_0] + x[n_1]\delta[n-n_1] + \ldots \rightarrow x[n_0]h[n-n_0] + x[n_1]h[n-n_1] \ldots$

    $\rightarrow \sum^{1}_{k=-1} x[k]h[n-k]$

    (The Convolution Sum!)