Solutions for the Quiz on Discrete-Time Frequency

A companion to the Discrete-Time Frequency demo. Use the applet with frequency selections of the form

$$\omega = (m/8)2\pi, \quad m = 0, 1, \ldots$$

to address the following questions. In some cases the answer requires mathematical manipulation of the expression for frequency.

1. Among the frequencies corresponding to $m = 0, 1, \ldots, 7$, what is the highest frequency?

At the frequency $\omega = \pi$, corresponding to $m = 4$, the phasor is rotating "as much as possible" at each step. (Also note that the real part is oscillating as fast as possible. Since the initial ($n = 0$) phasor angle is zero, the imaginary part is always zero. For a nonzero initial angle, the imaginary part would also oscillate as fast as possible.)

2. Why is the phasor signal the same for $m = 1$ and $m = 9$?

The corresponding frequencies differ by 2π, specifically, $(9/8)2\pi = (1/8)2\pi + (8/8)2\pi$.

3. Explain in mathematical terms why the real part of the phasor signal is the same for $m = 2$ and $m = 6$.

A trigonometric identity gives $\cos[(\omega + \pi)n] = (-1)^n \cos(\omega n)$, and for the two specified frequencies $\cos(\omega n) = 0$ for odd n.

4. What is the effect of changing the sign of the frequency, that is, changing the sign of the integer m?

The real part of the phasor is unchanged, while the sign of the imaginary part changes. In particular, note that the apparent "direction of rotation" does not necessarily change when the sign of the frequency changes – in contrast to the continuous-time case.